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Information sampling behavior with explicit sampling costs

Mordechai Z. Juni¹, Todd M. Gureckis², and Laurence T. Maloney^{2,3}

¹Department of Psychological and Brain Sciences, University of California, Santa Barbara

²Department of Psychology, New York University

³Center for Neural Science, New York University

Abstract

The decision to gather information should take into account both the value of information and its accrual costs in time, energy and money. Here we explore how people balance the monetary costs and benefits of gathering additional information in a perceptual-motor estimation task. Participants were rewarded for touching a hidden circular target on a touch-screen display. The target's center coincided with the mean of a circular Gaussian distribution from which participants could sample repeatedly. Each “cue” — sampled one at a time — was plotted as a dot on the display.

Participants had to repeatedly decide, after sampling each cue, whether to stop sampling and attempt to touch the hidden target or continue sampling. Each additional cue increased the participants' probability of successfully touching the hidden target but reduced their potential reward. Two experimental conditions differed in the initial reward associated with touching the hidden target and the fixed cost per cue. For each condition we computed the optimal number of cues that participants should sample, before taking action, to maximize expected gain. Contrary to recent claims that people gather less information than they objectively should before taking action, we found that participants over-sampled in one experimental condition, and did not significantly under- or over-sample in the other. Additionally, while the ideal observer model ignores the current sample dispersion, we found that participants used it to decide whether to stop sampling and take action or continue sampling, a possible consequence of imperfect learning of the underlying population dispersion across trials.

Keywords

information gathering behavior; optimal stopping; cost-benefit analysis; ideal observer model; sensitivity to sample dispersion

Human decision makers face a challenge: how to balance the advantage obtained from gathering additional information against the time, energy and money spent acquiring it. Stigler (1961) analyzed the economic costs of prolonging a search for a better price on a commodity. Since a relatively cheap price is often obtained after a relatively brief search, the cost of an exhaustive search will often not offset the savings gained by finding the cheapest price. Instead, consumers should continue searching only so long as the expected savings

from finding a cheaper price are enough to offset the additional costs in prolonging the search. Similar ideas are reflected in the mathematical literature on optimal sampling and optimal search (Wald, 1945a, 1945b; Arrow, Blackwell, & Girshick, 1949; Stone, 1989), and feature prominently in the study of animal foraging (Stephens & Krebs, 1986). The present study explores how effective humans are in trading off the costs and benefits of gathering additional information in a perceptual-motor estimation task with explicit sampling costs.

Prior Work On Information Sampling Behavior

Information gathering (or “sampling”) behavior was a topic of interest starting in the 1960s (e.g., Green, Halbert, & Minas, 1964; Edwards, 1965; Wendt, 1969; Rapoport & Tversky, 1970). Tversky & Edwards (1966), for example, had participants perform a probability-learning task. On each of 1000 trials, participants had to guess which of two mutually exclusive alternatives would occur. Although participants were rewarded at the end of the experiment for all of their correct guesses, they received no feedback following each guess. To gain information about the relative frequencies of the mutually exclusive alternatives, participants could forgo guessing on any particular trial and observe the outcome instead. They forfeited any possible reward on trials that they chose to observe which event would occur instead of guessing.

The optimal strategy maximizing expected gain in that study was to observe a certain number of trials at the start of the experiment, keep track of the relative frequencies of the mutually exclusive alternatives, and then choose on the remaining trials the alternative that had occurred more frequently (Wald, 1947). Participants in Tversky & Edwards (1966) greatly over-sampled information, observing around 300 trials spread out throughout the experiment, compared to the optimal sampling strategy of observing fewer than 33 or 38 trials (depending on the condition) at the start of the experiment. Busemeyer & Rapoport (1988) investigating a similar task also reported over-sampling in some experimental conditions.

Many recent studies, using card-sampling tasks for example, have not reported over-sampling however (e.g., Hertwig, Barron, Weber, & Erev, 2004; Hau, Pleskac, Kiefer, & Hertwig, 2008; Ungemach, Chater, & Stewart, 2009). Participants in these studies were presented with two decks of cards that had different, unknown payoff distributions. To gain information about the payoff distribution of each deck, participants were encouraged to freely sample as many cards as they wished “until they felt confident enough to decide from which [deck] to draw for a real payoff” (Hertwig et al., 2004). The single random draw for a real payoff at the end of the task was in effect the execution of a lottery where the probabilities of different monetary rewards was initially unknown but could be learned by sampling.

A common conclusion of these recent studies was that the median number of cards that participants sampled across both decks (between 11 and 19 cards depending on the study) was surprisingly small. Sampling more cards would lead to better estimates of the respective payoff distributions of the decks (Hertwig et al., 2004; Hau et al., 2008; Ungemach et al., 2009).¹

A limitation of these recent studies, however, is that the experimental designs could not be used to test whether implicit costs of sampling (e.g., the participant's time, boredom, etc.) could explain participants' sampling behavior. If there were absolutely no costs to sampling, then the optimal strategy maximizing expected gain would be to continue sampling indefinitely (or until the decks are depleted). No matter how many cards one has sampled already, additional samples lead to better estimates of the respective payoff distributions of the decks. *Compared to the optimal sampling strategy, participants could only under-sample.* On the other hand, if the costs of sampling were large enough, then the optimal behavior would correspond to not sampling at all, simply picking one of the decks at random without learning anything about each deck's payoff distribution and expected reward. *In such a case, any sampling at all would count as over-sampling.*

It is possible that participants in these studies stopped sampling cards because of implicit sampling costs.² But because we do not know what those costs are, it is incorrect to conclude that they under-sampled.

The goal of the present study was to examine information sampling behavior in a task where participants incur explicit sampling costs and explicit rewards for being successful and the optimal sampling rule is amenable to mathematical analysis. We will show in the Discussion that – by comparison of performance across experimental conditions – we can test whether implicit costs or rewards could explain participants' sampling behavior. This approach allows us to characterize participants' information sampling behavior: i.e., whether they over-sampled, under-sampled, or sampled optimally relative to a well-defined and realistic ideal observer model (Geisler, 2003).

Overview of the Current Experiment

Participants performed a perceptual-motor estimation task similar to those used by Battaglia & Schrater (2007) and Tassinari, Hudson & Landy (2006). The goal on each trial was to touch a hidden circular target on a touch-screen display to earn points. The target's radius remained constant throughout the experiment. The target's location changed randomly from trial to trial but was cued by dots drawn from a circular Gaussian that was centered on the middle of the hidden target. We refer to these dots as “cues”. The pointing strategy that maximizes the probability of touching the hidden target is to touch the screen at the center of gravity of the cues (i.e., the sample mean).

Participants were given the option to sample cues – one at a time – at a fixed cost per cue. The more cues the participant sampled, the greater the probability that the sample mean would fall within the target region (because the variance of the sample means decreases with increased sample size) and the greater the probability that the participant would successfully

¹The frequent reports of participants taking a small number of samples in recent studies have led a number of papers to argue that taking a small number of samples can be advantageous in certain situations (Todd, 2007; Vul et al., 2014). For example, Hertwig & Pleskac (2010) present an analysis showing that taking a small number of samples may amplify differences between the payoff distributions of different options (cf. Kareev, 2000) thus making it easier to choose between them (meeting the criterion of *satisficing* laid out by Simon, 1956).

²Another possibility is that participants stopped sampling cards because of working memory limitations (Hertwig et al., 2004). There is little point in sampling additional cards if the participant can't use them to update his or her estimate of the deck's payoff distribution and expected reward.

touch the hidden target. Previous studies with such tasks have shown that participants correctly interpret the arrival of more cues as reducing external uncertainty about the target's location, and that they are more successful in localizing the hidden target with more cues (Battaglia & Schrater, 2007; Tassinari et al., 2006).

In the current experiment, however, the benefit of sampling additional cues to reduce external uncertainty about the target's location came at a fixed monetary cost. The critical question is whether participants sample the optimal number of cues before taking action (i.e., before attempting to touch the hidden target): participants had to balance the benefit of gathering additional information (more cues) against the cost incurred to obtain it (Fu & Gray, 2006; Dieckmann & Rieskamp, 2007).

There is a growing interest in exploring different forms of decision making where probabilities arise from different sources, perceptual judgments, motor acts, or explicitly given numerical values (Trommershäuser et al., 2008; Wu et al., 2009; Maloney & Zhang, 2010; Summerfield & Tsetsos, 2012; Trueblood et al., 2013). If under-sampling behavior were a robust aspect of human judgment and decision-making, we would expect to see it emerge in many types of tasks beyond those commonly employed in the cognitive decision-making literature. Thus, our paper represents an attempt to generalize past findings to a wider variety of tasks.

Experimental Task

The participant's goal was to collect points by touching an invisible circular target placed at random on a touch-screen display. To gain information about the location of the hidden target, participants could request to observe the outcome of a theoretical dart-thrower who could see the hidden target (unlike the participants) and who would throw a dart towards it, aiming for its center. Participants could use the repeated outcomes of this theoretical person's repeated throws as cues to the location of the hidden target.

Each participant alternated between blocks of two different conditions that varied in the cost-benefit structure of the task: (1) a low stakes condition and (2) a high stakes condition. In the low stakes condition, the reward for touching the hidden target was initialized at 40 points on each trial and decreased by 2 points for every cue that was requested. In the high stakes condition, the reward for touching the hidden target was initialized at 60 points on each trial and decreased by 6 points for every cue that was requested.

Figure 1 illustrates two examples of the sequence of events within a trial. In the schematic example of a low stakes trial (left-hand panels), the participant chose to stop sampling and attempt to touch the hidden target after requesting nine cues. The bottom panel shows that the participant successfully touched the hidden target (the square response dot marking the location that the participant touched the screen is within the target region), and the target is revealed for feedback. The reward for this trial is 22 points, as indicated numerically at the center of the target. In the schematic example of a high stakes trial (right-hand panels), the participant chose to stop sampling and attempt to touch the hidden target after requesting four cues. The bottom panel shows that the participant missed the hidden target (the square

response dot marking the location that the participant touched the screen is outside the target region), and the target is revealed for feedback. Participants did not have to pay for the cues that they sampled on trials that they failed to touch the hidden target. Hence, the reward for this trial is 0 points (i.e., no reward), as indicated numerically at the center of the target.

Participants had to request at least one cue per trial, and they were allowed to continue sampling, one cue at a time, so long as the potential reward for successfully touching the hidden target remained greater than zero. Hence, they were limited to sampling 9 cues per trial in the high stakes condition and 19 cues per trial in the low stakes condition. The two experimental conditions were designed so that the maximum expected gain of the ideal participant (see below) would be very similar in the two conditions: 18.48 points per trial in the high stakes condition and 18.63 points per trial in the low stakes condition. The ideal participant obtains these maximum expected gains by sampling 4 cues on every trial in the high stakes condition and 7 cues on every trial in the low stakes condition (see derivation of the optimal sampling rule below).

What Is the Optimal Sampling Rule?

In order to analyze behavior in our task, we begin by defining an ideal participant who samples information with the goal of maximizing expected gain. The radius of the hidden circular target remained constant throughout the experiment. The underlying bivariate Gaussian distribution from which the cues were drawn was isotropic (“circular”) with equal variance in each direction: $\sigma_x^2 = \sigma_y^2$. For convenience, we refer to this common variance as *the population variance*, denoted σ_p^2 . The population variance remained constant throughout the experiment.

To maximize expected gain, an optimal participant must minimize their estimation variance by using the center of gravity of the cues (i.e., the sample mean) as their estimate of the hidden target’s location. The sample mean is the minimum variance unbiased estimate of the Gaussian population mean (Mood, Graybill, & Boes, 1974) which coincides with the center of the hidden circular target. Hence, for any number of cues, pointing at the sample mean is the strategy that maximizes the probability of touching the hidden target. The sample mean is distributed as an isotropic Gaussian centered on the target, but with reduced variance in each direction: σ_p^2/n . The larger the sample size n the more accurate the participant’s estimate of the center of the target.

Besides the variability of the sample mean, an optimal participant must take into account other sources of variability such as their perceptual-motor error in touching the screen. We refer to the aggregate of all the other sources of variability as *additional pointing variability*, and we will assume for simplicity throughout our analyses that it is isotropic Gaussian (Trommershäuser et al., 2003) and that its vertical and horizontal variances are both equal to a value denoted σ_A^2 . Together, these two sources of variance give rise to the participant’s *effective estimation variance* in each direction as a function of sample size, denoted $\sigma_E^2(n)$, as follows:

$$\sigma_E^2(n) = \sigma_P^2/n + \sigma_A^2. \quad (1)$$

In deriving the optimal sampling rule maximizing expected gain, we will first assume for simplicity that additional pointing variability is zero: $\sigma_A^2=0$. In the Discussion, we will revisit this assumption and explore the implications on optimal sampling behavior if $\sigma_A^2>0$. We will also assume for now that the ideal participant has an accurate estimate of the population variance. During the practice session prior to the experimental trials, the participant is given ample opportunity to develop an accurate estimate of σ_P^2 , by observing a very large number of draws from the underlying Gaussian as described below. In the Discussion, we will revisit this assumption as well and explore the implications on optimal sampling behavior if participants do not have an accurate estimate of σ_P^2 . Finally, because the target was revealed during feedback at the end of every trial (including during the practice session), we will assume for simplicity throughout our analyses that the participant has an accurate estimate of the radius of the hidden circular target.

Given $\sigma_E^2(n)$, we can compute the probability of touching the hidden circular target as a function of sample size as follows:

$$p[hit|n] = \iint_T \phi(\vec{0}, \Sigma(n)) dx dy, \quad (2)$$

where the region of integration T is the invisible circular target, and $\phi(\vec{0}, \Sigma(n))$ denotes the probability density function of a bivariate Gaussian distribution centered on $\vec{0}=(0,0)$ with covariance

$$\Sigma(n) = \begin{bmatrix} \sigma_E^2(n) & 0 \\ 0 & \sigma_E^2(n) \end{bmatrix}. \quad (3)$$

We can estimate $p[hit|n]$ by computing the probability that a draw from an isotropic Gaussian distribution, whose variance in each direction matches $\sigma_E^2(n)$, will fall within the circular target region. This depends only on the probability that the distance between the drawn point and the origin is less than the radius of the target. This distance squared is a χ_2^2 random variable multiplied by $\sigma_E^2(n)$, and we need only compute the cutoff corresponding to the target radius squared to estimate the probability of a hit.

Given $p[hit|n]$, we can compute the expected gain as a function of sample size as follows:

$$EG(n) = p[\text{hit}|n](R - nC), \quad (4)$$

where R is the initial point value of the target, and C is the fixed reduction to the target value that is incurred for each cue that is sampled. Participants were not penalized for missing the target, and they were allowed to sample additional cues only so long as $(R - nC) > 0$. All plots will be cut off where $EG(n)$ starts to dip below zero.

Figure 2 plots the gain function $R - nC$, the probability of hitting the target $p[\text{hit}|n]$, and their product, which is the expected gain function $EG(n)$, against the number of cues sampled for the low stakes condition. The expected gain function is greatest when $n = 7$. The ideal participant who seeks to maximize expected gain would sample 7 cues and then touch the mean of the sampled cluster. A similar construction (see middle panel of Figure 7A and top panel of Figure 7B) shows the optimal number of cues that maximizes expected gain for the high stakes condition: $n = 4$.

Dispersion

The ideal participant derived above makes its decision whether to stop or continue sampling without reference to how spread out a sampled cluster is. Since the ideal model has an accurate estimate of the population variance σ_p^2 , there is no need for the ideal participant to make use of any aspect of a sampled cluster beyond its mean (just as, in computing a z-test with known population variance, we make no use of sample variance). We will revisit this assumption in the Discussion and it will prove convenient to define a measure for how spread out a sampled cluster is. While the underlying bivariate Gaussian was isotropic, the vertical and horizontal variances of a sampled cluster are usually different from each other and from one cluster to the next because of normal sampling variability. We define *sample dispersion* as $S = \sqrt{(S_x^2 + S_y^2) / 2}$, where S_x^2 and S_y^2 are the respective horizontal and vertical variances of a sampled cluster. The corresponding *population dispersion* is simply σ_p , the square root of the population variance σ_p^2 .

Method

Participants and Apparatus

13 women and 15 men participated one at a time in the experiment, for a total of 28 participants. Participants were graduate students, postdocs, or lab managers between the ages of 23 and 43, with a median age of 26 and a mean age of 27.6 (SD=4.71). None were aware of the purpose of the experiment. Participants were paid \$10 plus any monetary bonus that they earned, which depended on their performance.

Eight participants were run at New York University, and 20 participants were run at the University of California, Santa Barbara. All conditions were identical except that the touch screen used at UCSB (19 inch diagonal) was larger than the one used at NYU (17 inch diagonal). Both LCD touch screens had the same resolution of 1280 pixels by 1024 pixels,

and we report all measurements in pixels. The conversion factor from pixels to millimeters is 0.293 for the UCSB touch screen (375 mm by 300 mm) and 0.2637 for the NYU touch screen (337.5 mm by 270 mm).

Participants were asked to seat themselves at a comfortable distance and to adjust the height of the chair. The experiment was programmed and run using MATLAB and the Psychophysics Toolbox libraries (Brainard, 1997; Pelli, 1997).

Stimuli

The hidden target, which was revealed during feedback at the end of every trial, was a grey circle with radius = 48 px. The cues, which remained on the screen throughout the trial, were small white circular dots with radius = 4 px. The cues were drawn from an isotropic, bivariate Gaussian distribution ($\sigma_x = \sigma_y = \sigma_p = 80\text{px}$) that was centered on the middle of the hidden target. At the left and right edges of the screen there were identical, vertical reward bars that decreased in height with each additional cue sampled (see schematic in Figure 1). Additionally, there was a number at the top of each bar indicating how many points would be awarded for successfully touching the target if no additional cues are sampled. To help participants remember which condition they were in, the reward bars were colored green during low stakes trials and red during high stakes trials.

Design

Each participant ran in a practice session followed by the experimental session. The practice session consisted of 30 trials of each condition in alternating blocks of 10 trials, for a total of six practice blocks and 60 practice trials. The actual experiment consisted of 100 trials of each condition in alternating blocks of 25 trials, for a total of 8 experimental blocks and 200 experimental trials. The ordering of the conditions was randomly assigned and counterbalanced between participants. However, the ordering was kept constant between the practice session and the experimental session. Hence, a participant assigned to start with the high stakes condition would start both the practice and the experimental sessions with a high stakes block. Each participant ran alone and the task was self-paced. Participants' participation time (including practice) ranged from 34 min to 74 min, with an average of 53 min.

Monetary bonus

Points earned were converted into bonus money at a rate of five cents per point. This means that the maximum possible reward for touching the target was $38 \text{ points} \times \$0.05 = \$1.90$ per low stakes trial, and $54 \text{ points} \times \$0.05 = \$2.70$ per high stakes trial. Participants were informed that they would receive a monetary bonus on 5% of the trials, randomly chosen from the trials they completed in each condition at the end of the experiment (5 random trials from each condition were selected for payment). The total expected monetary bonus of the ideal participant who maximizes expected gain was: $10 \text{ trials} \times 18.55 \text{ points} \times \$0.05 = \$9.28$.

Procedure

Participants were told that the theoretical dart-thrower was standing behind the participant at some unknown distance away from the touch-screen display. They were told that this theoretical person could see the hidden target (unlike the participants) and aimed each dart-throw at the center of the target, but that when people throw darts, they rarely hit exactly where they aim. The experimenter emphasized that the theoretical dart-thrower was always going to throw the dart in the same way, always aiming for the center of the target, and always standing in the same exact location and at the same exact distance away from the target. Participants were explicitly told that this theoretical person's throwing ability never improved or deteriorated throughout the experiment. Finally, participants were explicitly told that the size of the circular target would remain the same throughout the experiment.

Participants were asked to use their dominant hand throughout the experiment. The experimenter stayed in the room during the practice blocks to explain the task. The experimenter first demonstrated how one could sample as few as one cue per trial, and as many as 9 and 19 cues per trial (depending on the condition). The experimenter then told the participant that he or she should use the practice trials to explore and observe the outcome of different decision strategies. The experimenter remained in the room during the practice session to answer any questions that came up. Once the practice trials were complete, the experimenter left the room closing the door behind him.

At the start of each block, participants were shown an instruction screen that specified the initial point value of the target on each trial (40 points or 60 points) and the cost per cue (2 points or 6 points). Participants were required to confirm the appropriate cost per cue by pressing "2" or "6" before the block would begin.

At the start of each trial, the screen was completely black except for the reward bars, which indicated the current reward for touching the target. To sample a cue, participants pressed the spacebar causing a small white circular dot to appear on the screen (the schematic in Figure 1 shows the cues as small black circular dots on a white background). Concurrent with the appearance of the cue, the reward bars and numbers would decrease to indicate the decreased reward available for touching the target. If participants wanted more cues, they simply pressed the spacebar repeatedly until they were ready to reach for the hidden target. This easy and rapid mechanism of requesting cues and re-computing the value of hitting the target ensured that the information-access costs associated with sampling information were minimized.

To attempt to touch the invisible target, participants simply touched the screen with their finger causing a small purple response dot to appear (the schematic in Figure 1 shows the response dot as a black square dot). Participants were allowed to adjust their response by moving the response dot with their finger or by pressing the arrow keys. Once satisfied with their response, participants pressed the spacebar to receive feedback. During the feedback phase of each trial, the hidden target would appear along with the response dot and all the cues sampled on that trial. If the response dot was within the target, the trial was counted as a hit and the number of points awarded for that trial would appear at the center of the target (see left-hand side of Figure 1). If, however, the response dot was not within the target, then

the trial was coded as a miss and a 0 appeared at the center of the target (see right-hand side of Figure 1). A second spacebar press began the next trial.

Finally, at the end of both the practice session and the experimental session, participants were given feedback as to how they performed on each of 10 randomly selected experimental trials and how much bonus money they received as a result. The feedback at the end of the practice session was not actually paid out, but served to give the participant a rough sense of the range of actual bonuses possible.

Results

1. Number of Cues Sampled

The solid curves in Figure 3 show the respective high stakes and low stakes expected gain functions for the ideal participant with additional pointing variability equal to zero ($\sigma_A^2=0$). The expected gain for this ideal participant is maximized when sampling 4 cues in the high stakes condition and 7 cues in the low stakes condition. The horizontal displacement of data points in Figure 3 shows each participant's mean and standard deviation of the number of cues sampled for each of the two experimental conditions. In addition, the individual data points appear at heights that correspond to their average gain per trial. For similar sampling behavior (i.e., similar location along the x-axis), some participants were better able to successfully touch the hidden target and collect more points (i.e., their data points are located higher up along the y-axis). The average number of points per trial that each participant earned across both experimental conditions ranged from 9.04 to 18.75 with a mean of 14.03 (SD=2.08), which is 76% of the maximum expected gain of 18.55 points per trial.

1.1 Participants Adapted to Changes in the Cost-Benefit Structure of the Task—Overall, participants sampled significantly more cues per trial in the low stakes condition (M=6.94, SD=1.65) compared to the high stakes condition (M=5.65, SD=1.05), $t(27)=6.22$, $p<.001$. This indicates that participants adapted to changes in the cost-benefit structure of the task. Individually, 24 participants sampled significantly more cues per trial in the low stakes condition than in the high stakes condition ($p<.05$), one sampled significantly more cues per trial in the high stakes condition than in the low stakes condition ($p<.05$), and three showed no significant difference in the number of cues sampled per trial between conditions ($p>.05$); all individual significance tests were t-tests with 99 degrees of freedom. This result rules out the possibility that participants weren't doing any cost-benefit analysis at all, and that they were only concerned with the challenge of touching the hidden target, a challenge that is identical in both conditions (up until nine cues, which is the limit in the high stakes condition).

1.2 Participants Did Not Exhibit Systematic Under-Sampling—Overall, participants, did not significantly differ from sampling the ideal seven cues per trial in the low stakes condition (M=6.94, SD=1.65, $t(27)=0.18$, $p>.05$). Individually, 11 participants sampled significantly more than seven cues per trial ($p<.05$), 12 sampled significantly less than seven cues per trial ($p<.05$), and five did not significantly differ from sampling seven

cues per trial ($p > .05$); all individual significance tests were t-tests with 99 degrees of freedom.

On the other hand, participants sampled significantly more than the ideal four cues per trial in the high stakes condition ($M=5.65$, $SD=1.05$, $t(27)=8.35$, $p < .001$). Individually, 26 participants sampled significantly more than four cues per trial ($p < .05$), one sampled significantly less than four cues per trial ($p < .05$), and one did not significantly differ from sampling four cues per trial ($p > .05$); all individual significance tests were, as above, t-tests with 99 degrees of freedom.

Taken together, participants did not exhibit any systematic tendency to under-sample information, and actually over-sampled information relative to ideal in the high stakes condition.

2. Variability in Sampling Behavior Across Trials

Because of normal sampling variability, the currently visible cluster is sometimes more spread out and sometimes less spread out. The ideal observer model described above has an accurate estimate of the underlying population dispersion σ_P and should pay no attention to the sample dispersion S of the currently visible cluster (defined above). Specifically, because the cues are independent and identically distributed draws from a Gaussian distribution, the *variance of the sample means* in each direction, which is equal to σ_P^2/n , is the same for clusters of size n that happen to have low sample dispersion as for clusters of the same size n that happen to have high sample dispersion. Consequently, the probability of touching the hidden target $p[\textit{hit}|n]$, which is maximized by pointing at the mean of the sampled cluster, is the same for high-dispersion clusters of size n as for low-dispersion clusters of the same size n . We verified this fact through computer simulation. The only factor that affects the variance of the sample means (besides for the sample size n) is the underlying population dispersion σ_P , which was constant throughout the experiment.

In the high stakes condition, an optimal decision maker will always request four cues irrespective of how dispersed the cues are on the screen, while in the low stakes condition they will always request seven cues. Nonetheless, almost all participants in our experiment exhibited variability in the number of cues that they requested from one trial to the next (see Figure 4 for each participant's trial by trial sampling behavior). Combined with our finding of over-sampling in the high stakes condition (see above), this result reveals a second way in which the participants' sampling behavior differed from optimal.

One possible explanation is that variation in sampling behavior from trial to trial is the result of unsystematic noise in each individual's decision-making process. A second possibility, which we evaluate below, is that some random characteristics of the visible cues during each trial, specifically their sample dispersion S (which is an index of how spread out the current cluster is), influenced the participants' decision whether to stop or continue sampling, inducing variability in sampling behavior across trials.³

³We considered several different hypotheses for this analysis including (a) whether participants used the area of the convex hull of the currently visible cluster to guide their sampling behavior, (b) whether participants used the outlier-ness of the current cue relative to

2.1 Stop Clusters vs. Continue Clusters—During a trial, participants sampled cues one at a time and had to decide, after sampling each cue, whether to stop sampling and attempt to touch the hidden target (given the currently visible cluster) or to continue sampling. The cumulative number of cues sampled so far during the trial (i.e., the current sample size) can be represented by the symbol N . In this section, we computed the sample dispersion S of the currently visible cluster for all values of N , and compared said sample dispersion when participants stopped sampling after the N th cue to when they continued sampling after the N th cue.

Table 1 shows the total number of “stop” clusters and “continue” clusters across all participants as a function of N in each experimental condition. For example, participants almost always continued sampling after seeing only one cue (the exception being just one case in the high stakes condition). At the other extreme ($N=9$ in the high stakes condition, and $N=19$ in the low stakes condition), participants were obligated to stop sampling and take action (i.e., attempt to touch the hidden target). As a result, these two extremes were excluded from this analysis. For intermediate values of N , sometimes there were enough stop clusters and continue clusters to conduct this analysis meaningfully, but other times there were not. As such, we have shaded the cells in Table 1 for which we decided not to conduct a comparison because of the limited number of stop clusters and/or continue clusters. Overall, we carried out this analysis 16 times across both conditions (see the cells in Table 1 that are not shaded), and readers should keep in mind a Bonferroni correction of $.05/16 = .0031$ when considering whether any particular result is statistically significant.

As shown in Table 1, there were 2641 trials in which participants sampled at least four cues in the high stakes condition. In 464 of those trials they decided to stop sampling and take action, while in the other 2177 they decided to request another cue. In Table 2 (high stakes condition; $N=4$) we show that the currently visible cluster when participants chose to continue sampling had a mean sample dispersion of 78.05 px ($SD=21.86$), compared to a mean sample dispersion of 71.65 px ($SD=23.43$) when they chose to stop sampling and attempt to touch the hidden target. A statistical test indicates that this difference in mean sample dispersion was significant: $t(2639)=5.66, p<.001$. Likewise, there were 2177 trials in which participants sampled a fifth cue in the high stakes condition. In 704 of those trials they decided to stop sampling and take action, while in the other 1473 they decided to continue sampling. The mean sample dispersion for continue clusters ($M=80.72$ px, $SD=19.03$) was significantly larger than the mean sample dispersion for stop clusters ($M=74.72$ px, $SD=19.03$), $t(2175)=6.86, p<.001$.

Table 2 shows the respective mean sample dispersion for “stop” clusters and “continue” clusters as a function of N in each experimental condition. Statistical tests are shown in bold when the mean sample dispersion was significantly larger for continue clusters than for stop clusters at the Bonferroni corrected p of .0031. The table shows that for almost all values of

the previous cues of the currently visible cluster to guide their sampling behavior, and (c) whether participants used the sample dispersion S of the currently visible cluster to guide their sampling behavior. We give a full account of the analysis of this last hypothesis because it was the best predictor of participants’ sampling behavior among those we considered.

N, and irrespective of the experimental condition, continue clusters tend to have higher sample dispersion than stop clusters.

These observations suggest that participants were influenced by how spread out the currently visible cluster was in deciding whether to stop sampling and attempt to touch the hidden target or to continue sampling. The larger the sample dispersion S , the more likely it is that participants would continue to sample.

2.2 High Dispersion Clusters vs. Low Dispersion Clusters—For each value of N, and separately for each experimental condition, we sorted all trials across all participants based on the sample dispersion S of the currently visible cluster and grouped them into three equal-sized groups: high dispersion clusters, medium dispersion clusters, and low dispersion clusters. When the total number of clusters for a particular value of N was not divisible by three, we assigned an equal number of clusters to the high- and low-dispersion groups, with the surplus assigned to the medium-dispersion group.

The analysis of most interest in this section is the proportion of clusters that participants chose to continue sampling, denoted P[continue]. The circle data points in Figure 5 show P[continue] across all clusters, irrespective of their sample dispersion, as a function of N in each experimental condition. Notice that P[continue] tends to decrease as the number of cues sampled increases. This pattern is expected given the costs of requesting additional cues. However, as also shown in the figure, P[continue] in both conditions is consistently greater for clusters with high sample dispersion than for clusters with low sample dispersion, with P[continue] for clusters with medium sample dispersion nestled in between.

This analysis supports our conclusion from the previous section that increased sample dispersion S seems to prompt a greater tendency to continue sampling. For example, in the high stakes condition the proportion of clusters that participants chose to continue sampling for N=6 was 39% greater for clusters with high sample dispersion (P[continue] = .57) than for clusters with low sample dispersion (P[continue] = .41).

Discussion

Overall, our results indicate that participants correctly sampled more cues in the low stakes condition compared to the high stakes condition. However, the results also show two distinct ways in which participants diverged from optimal sampling behavior. First, in the high stakes condition – but not the low – participants requested more cues relative to the optimal sampling rule that maximizes expected gain. Second, participants varied the number of cues that they requested from one trial to the next. We demonstrated that this trial-to-trial variation was correlated with sample dispersion: when the currently visible cluster was more widely dispersed participants were more likely to request another cue. But for the ideal participant maximizing expected gain, the sample dispersion of the current cluster – as opposed to the fixed dispersion of the underlying Gaussian – should play no role in deciding whether to stop sampling and attempt to touch the hidden target or to continue sampling. In the following sections we consider a range of hypotheses that might explain the observed patterns of sampling behavior.

3. Possible Explanations of Participants' Over-Sampling Behavior

3.1 Asymmetry Around the Peak of the Expected Gain Function—The expected gain functions in Figure 3 are asymmetric around the maximum. One hypothesis is that participants selected a cautious sampling strategy that is biased towards the shallower side of the expected gain function, to avoid the risk of ending up on the steeper side. Such a strategy of “erring on the shallower side” would lead to over-sampling in our case.

We doubt, however, that the observed over-sampling can be explained in this way. First, the keyboard presses to request cues were discrete, readily distinguishable, and completely under the participant's control. There is no need to cautiously plan to press the sampling key 5 times, instead of the optimal 4, to avoid the negligible risk of pressing it only 3 times. Second, the difference in expected gain between the shallower side (over-sampling) and the steeper side (under-sampling) is smaller than the difference in expected gain between both sides and the peak of the expected gain function. If participants, in the high stakes condition for example, were sensitive to the difference in expected gain between sampling 5 cues (17.80 points) and 3 cues (17.52 points) and thus over-sampled to err on the shallower side of the expected gain function, then they should also be sensitive to the larger difference in expected gain between the optimal 4 cues (18.48 points) and 5 cues (17.80 points).

3.2 Optimal Compensation for Additional Pointing Variability—In deriving the optimal sampling rule, we assumed for simplicity that additional pointing variability (due to motor error, perceptual error, etc.) was negligible ($\sigma_A^2=0$): i.e., that the participant is perfectly accurate in touching the mean of the sampled cluster. One hypothesis is that participants sampled additional cues (relative to an ideal observer who has negligible pointing variability) to reduce external uncertainty about the target's location as an attempt to compensate for additional variability in their perceptual-motor pointing response.

Sampling additional cues when $\sigma_A^2 > 0$, however, actually reduces the participants' expected gain relative to the maximum expected gain that they could achieve given any additional pointing variability. Figure 6 shows that as additional pointing variability increases, the ideal observer would sample *fewer* cues – not more – in order to maximize expected gain. To see why, consider an extreme case where an individual's perceptual-motor pointing error is so high that the chances of even touching the screen are very small. In such a case, the reduced external uncertainty in localizing the hidden target that is obtained by sampling additional cues is negligible compared to the individual's extreme pointing variability. Instead, it would be best to just try to touch the screen (aiming for the middle of the screen) without sampling any cues at all, so that on the lucky occasions that the hidden target is touched the reward will be great.

In summary, taking into account additional pointing variability (i.e., $\sigma_A^2 > 0$) would lead to a *decrease* in the number of cues one should optimally sample in order to maximize expected gain. However, as shown in Figure 3, most participants over-sampled. Hence, participants' over-sampling behavior cannot be explained as optimal compensation for any additional variability in their perceptual-motor pointing response.

3.3 Risk Aversion—Over-sampling cues in this task is reminiscent of common descriptions of risk aversion in the literature. We explored the implications of having a nonlinear utility function, specifically a power function of the form $U(x)=x^\alpha$ (Kreps, 1990, p. 82), rather than the linear gain function used in our ideal observer model. The gain function $R-nC$ in Eq. 4 is replaced by a power utility function: $U(R-nC)=(R-nC)^\alpha$.

When $\alpha>1$ the power utility function is strictly convex (“convex utility function”) and when $0<\alpha<1$ the power utility function is strictly concave (“concave utility function”). See Figure 7A. As shown in the top panel of the figure, a convex utility function ($\alpha=1.5$) shifts the optimal number of cues necessary to maximize expected utility in each condition to the left (i.e., fewer cues) relative to an ideal observer with a linear utility function (middle panel). Conversely, and as shown in the bottom panel of the figure, a concave utility function ($\alpha=0.5$) shifts the optimal number of cues necessary to maximize expected utility in each condition to the right (i.e., more cues).

The hypothesis, then, is that the observed over-sampling (relative to an ideal observer with a linear utility function) is a consequence of having a concave utility function. However, while over-sampling in any one experimental condition can be fit with a concave utility function, no single concave utility function can account for participants’ pattern of sampling behavior across *both* experimental conditions. We found (by simulation) that any single concave utility function for our task will generate a greater over-sampling of cues for the low stakes condition than for the high stakes condition. I.e., the rightward shift of the peak of the expected utility function, relative to the peak of an ideal observer with a linear utility function, will be greater for the low stakes condition than for the high stakes condition.

For example, and as shown in the bottom panel of Figure 7A, a concave utility function with $\alpha=0.5$ results in a rightward shift of 2 cues to the peak of the expected utility function for the low stakes condition (from 7 to 9) vs. a rightward shift of 1 cue for the high stakes condition (from 4 to 5). However, only 2 participants over-sampled more in the low stakes condition than in the high stakes condition. The rest of the 24 participants who over-sampled in the experiment (2 participants did not over-sample in either condition) either over-sampled *only* in the high stakes condition (15 participants), or over-sampled *more* in the high stakes condition than in the low (9 participants).⁴ Hence, participants’ over-sampling behavior cannot be explained as the consequence of having any single utility function that is a power function.

We also investigated (by simulation) the implications of having a nonlinear subjective probability function, using the model of Tversky & Kahneman (1992) as a reference. While nonlinear transformations of $p[\textit{hit}|n]$ alter the expected gain function $EG(n)$ and could account for participants’ sampling behavior when looking at any one experimental condition in isolation, we found no single subjective probability function that could account for participants’ pattern of sampling behavior across *both* experimental conditions.

⁴For example, one participant (NYU 1 in Figure 4) sampled an average of 6.83 cues per trial in the high stakes condition (optimal is 4) and 8.15 cues per trial in the low stakes condition (optimal is 7). Hence, this participant over-sampled an average of 2.83 cues per trial in the high stakes condition compared to 1.15 cues per trial in the low stakes condition.

3.4 Implicit Costs and Rewards—Our experimental paradigm allows us to test whether implicit costs or rewards could explain participants' sampling behavior. If participants have implicit rewards associated with success and also implicit costs associated with sampling, then only the sum of the two can be estimated from performance: i.e., the net implicit cost/reward. Had participants under-sampled (relative to an ideal observer who does not have any implicit costs or rewards) we would attempt to estimate the participants' net implicit costs to test whether they could explain participants' sampling behavior. Given that they did not under-sample, the participants' net implicit cost/reward must be positive or 0.

The hypothesis, then, is that participants experience an implicit reward for successfully touching the hidden target – independent of the actual points earned – and that this implicit reward leads them to over-sample (cf. Juni, Gureckis, & Maloney, 2012). We evaluated this hypothesis by putting a fixed point-value on the implicit reward for successfully touching the hidden target (a free parameter). This free parameter shifts the diagonal reward function $R-nC$ in Figure 2 upwards, which then causes a rightward shift to the peak of the expected gain function $EG(n)$, with the magnitude of the rightward shift to the peak of the expected gain function being controlled by the magnitude of the upward shift to the diagonal reward function.

However, no single value of implicit reward can account for participants' pattern of sampling behavior across *both* experimental conditions. We found (by simulation) that any fixed implicit reward for successfully touching the hidden target will generate a greater over-sampling of cues (relative to an ideal observer who does not have any implicit reward and shown in the top panel of Figure 7B) for the low stakes condition than for the high stakes conditions.

For example, and as shown in the bottom panel of Figure 7B, a fixed implicit reward of 20 points results in a rightward shift of 2 cues to the peak of the expected reward function for the low stakes condition (from 7 to 9) vs. a rightward shift of 1 cue for the high stakes condition (from 4 to 5). However, as mentioned in the previous section, 24 of the 26 participants who over-sampled in the experiment over-sampled more in the high stakes condition than in the low. Hence, participants' over-sampling behavior cannot be explained as the consequence of having any fixed implicit reward for successfully touching the hidden target.

4. Sensitivity to Sample Dispersion

Our results indicate that participants were influenced by how spread out the currently visible cluster was in deciding whether to stop sampling and attempt to touch the hidden target or to continue sampling. We next consider why participants were more likely to continue sampling when the currently visible cluster was more dispersed and less likely when it was less dispersed.

4.1 The Ideal Observer Model Has An Accurate Estimate of Population Dispersion—In deriving the optimal sampling rule, we assumed that the ideal participant has full knowledge of the underlying distribution from which the cues are drawn except for its mean (which coincides with the center of the hidden target). In particular, we assume that

the ideal participant knows the population dispersion σ_P of the underlying Gaussian. As explained in the Results, the ideal participant is insensitive to the incidental sample dispersion S of the current cluster, and will request the same number of cues on every trial regardless of how spread out the current cluster is. It is true that the dispersion of the current cluster provides information about the underlying population dispersion; but the ideal observer already knows the population dispersion. As a consequence, the ideal participant always samples the same number of cues on every trial (either 4 or 7 depending on the condition).

Participants could in principle get an accurate estimate of the population dispersion via the practice session prior to the experimental trials (the actual population dispersion σ_P of the underlying Gaussian was 80 px). The lowest cumulative number of cues sampled by any participant over the course of the 60 practice trials was 188 cues (the average across all 28 participants was 410 cues, and the maximum was 774 cues). A participant with perfect memory and computational power would use 188 random draws to estimate the population dispersion of the underlying Gaussian at an average of 79.95 px (SD=2.92). However, arriving at such an accurate estimate of this task parameter requires accumulation of information across many trials.

4.2 Hypothetical Memory-Less Model—Suppose, in an extreme case, that a hypothetical “memory-less” participant carries over no information about the underlying population dispersion across trials (though it is afforded knowledge that the cues are drawn from a circular Gaussian). In such a case, the participant could only use the information about cue dispersion that is available within the current trial to decide when to stop sampling (remember that the cues remained on the screen throughout the trial, so no memory is required to compute the dispersion of the current cluster). Every trial for such a hypothetical memory-less participant is, in effect, the first and only trial in terms of estimating the dispersion of the underlying distribution, which is in turn re-estimated with each additional cue that is sampled within a trial.

For such a memory-less model, a good estimate of population dispersion is the sample dispersion S of the current cluster. This estimated dispersion squared replaces the population variance σ_P^2 in Eq. 1 in estimating the effective estimation variance $\sigma_E^2(n)$ as a function of sample size n , and the consequent estimated probability of successfully touching the hidden target $p[\text{hit}|n]$ that is given by Eq. 2. Using this estimated $p[\text{hit}|n]$, the memory-less model uses Eq. 4 to determine whether the expected gain $EG(n)$ is higher or lower with more cues. If expected gain is estimated to decrease with more cues, then the memory-less model stops sampling and takes action (by pointing at the mean of the current cluster). If expected gain is estimated to increase with more cues, the memory-less model samples another cue and then repeats all these steps, starting with a re-estimation of population dispersion given the new cue that was added to the cluster.

We simulated a version of this hypothetical “memory-less” participant⁵ and summarize its hypothetical sampling behavior in Figure 8 and Table 3. The first thing to note is that while the ideal participant (with full knowledge of the underlying population dispersion) always

samples the same number of cues on every trial (either 4 or 7 depending on the condition), this hypothetical memory-less participant samples a variable number of cues across trials.

Table 3 shows that this memory-less participant's variable sampling behavior across trials qualitatively matches human behavior (see Table 2) as follows: For all values of N , and irrespective of the experimental condition, continue clusters for this memory-less participant have a (much) higher sample dispersion than stop clusters. Furthermore, the mean sample dispersion of the clusters increases (greatly) for this memory-less participant as the number of cues sampled increases.

In summary, the memory-less model stops sampling when the sample dispersion S of the current cluster is relatively low and continues sampling when it is relatively high. This pattern qualitatively resembles human behavior, though the difference in mean sample dispersion between “stop” and “continue” clusters is much more extreme for the hypothetical memory-less participant that carries over no information across trials about the underlying population dispersion.

We conjecture that a compromise model based on the assumption that the participant carries over partial information across trials about population dispersion, perhaps in the form of an updating prior distribution on population dispersion, might better match human variability in sampling behavior across trials. If this conjecture is correct, then the observed link between the number of cues sampled and the sample dispersion S of the current cluster would be due to imperfect learning of the underlying population dispersion across trials.

If, in future work, we were to abruptly change the dispersion of the underlying Gaussian after several hundred trials, we could likely observe the effect of the prior. The memory-less participant's sampling behavior would be affected by the new population dispersion *immediately*. The participant with a prior would lag behind until their prior is updated to agree with the new population dispersion, and the relative importance of prior and present might determine the rate of adaptation.

Finally, our analysis and simulation of the hypothetical “memory-less” participant shows that the observed human sensitivity to sample dispersion (cf. Table 2 and Figure 5) likely contributed to their trial-to-trial variation in the number of cues that they sampled. However, the hypothetical memory-less participant's slight tendency to under-sample relative to ideal (see Figure 8 and compare to the ideal 4 and 7 cues depending on the condition) is a *constant error* (bias) that can be readily corrected. A modified hypothetical memory-less model that corrects this bias would no longer on average under-sample (or over-sample) relative to ideal, but would still sample a variable number of cues across trials. That is, a sensitivity to sample dispersion does not, on its own, lead to over-sampling or under-sampling, but it likely does contribute to variability in the number of cues sampled across trials.

⁵As our participants almost never stopped sampling after seeing only one or two cues (cf. Table 1), we forced this simulated version of the memory-less model to sample at least three cues per trial.

Conclusion

We reported an experiment assessing how effective humans are in trading off the costs and benefits of sampling additional information in a perceptual-motor estimation task with explicit sampling costs. In our analyses we focused on the intermediate decisions that participants make after sampling each cue: *whether to stop sampling and take action or continue sampling to reduce uncertainty*. These intermediate decisions are similar to those in other cognitive decision-making tasks, such as the card-sampling tasks used in many recent studies (Hertwig et al., 2004; Hau et al., 2008; Ungemach et al., 2009; Hertwig & Pleskac, 2010).

Our results have relevance to a broad range of information sampling tasks. For example, when learning new concepts or categories, people can often choose to actively sample additional members of a group (Markant & Gureckis, 2014). In such cases, issues like stopping rules for terminating active sampling, the number of samples actively requested, and the need to balance information-gain against sampling-costs play central roles.

Our experimental design has several advantages. First, given the explicit costs of sampling and explicit rewards for being successful, we can compute the optimal number of cues that an ideal participant would sample before taking action to maximize expected gain. Second, we tried to minimize any implicit costs of sampling (e.g., the participant's time, boredom, etc.). The task was engaging; sampling a cue entailed pressing a key, which is a very simple and rapid action; and we limited the number of cues that participants could sample on every trial (either 9 or 19 depending on the condition). Third, our design allows us to test whether any observed deviations from optimal sampling behavior can be explained by implicit costs or rewards. This allows us to draw meaningful conclusions about how to characterize participants' sampling behavior relative to a well-defined and realistic ideal observer model (Geisler, 2003).

We found that in one condition (high stakes) participants sampled significantly more cues than that predicted by the ideal observer model, while in the other condition (low stakes) the number of cues that participants sampled was not significantly different from that predicted by the ideal observer model. In contrast to the predictions of recent theories intended to account for why people under-sample in cognitive decision-making tasks (Hertwig & Pleskac, 2010; Vul et al., 2014), we found no evidence of under-sampling in our task.

We explored a range of hypotheses to account for participants' over-sampling behavior in our experiment including (a) optimal compensation for any additional pointing variability (due to motor error, perceptual error, etc.), (b) risk aversion induced by any single concave utility function and (c) any fixed implicit reward for successfully touching the hidden target that is independent of the explicit reward earned. However, none of these hypotheses could account for the observed patterns of over-sampling behavior in our experiment.

The existing literature on information sampling behavior consists of experiments with designs that differ in many respects and it is difficult to isolate factors that might lead to over-sampling or under-sampling. Further studies are needed to determine the conditions under which people over-sample, under-sample, or sample optimally relative to ideal. We

emphasize however that it is difficult, if not impossible, to assess human sampling behavior relative to a realistic ideal without taking into account any *implicit* costs and rewards. In this study we demonstrated that this could be accomplished by comparing performance across different conditions with different *explicit* costs and rewards.

An additional novel finding from our study is that participants were more likely to continue sampling when the currently visible cluster was more dispersed and less likely when it was less dispersed. Since variance estimation is important in many decision-making problems (Kareev et al., 2002), this is likely an important factor to consider in studies of information gathering behavior.

Participants in our study were given ample opportunity during the practice session to learn the dispersion of the underlying Gaussian from which the cues were drawn. Given an accurate estimate of the population dispersion, the ideal sampling strategy maximizing expected gain ignores the current sample dispersion. Hence, sensitivity to sample dispersion in our experiment can only reduce the participant's expected earnings relative to the ideal observer model. However, if the participant for whatever reason does not have an accurate estimate of the population dispersion, then the optimal sampling strategy maximizing expected gain should take into account the current sample dispersion. We conjecture that, for whatever reason, participants did not accumulate an accurate estimate of the underlying population dispersion across trials, and instead relied in part on the current sample dispersion to decide whether to stop sampling and take action or continue sampling.

As an everyday analogy, imagine trying to estimate the overall typicality of a genre of music by paying to listen to random examples of the genre online. Instead of deciding in advance how many examples to pay for, it would make sense to assess after each example whether or not to pay for another sample. The listener would pay for more samples if the early examples sound very different from one another (heightened early variance suggests greater uncertainty), and would stop sampling if the early examples sound quite alike. As Edwards (1965) notes, "if a small set of observations happens to be sufficiently conclusive, there is no need to buy more" (p. 312).

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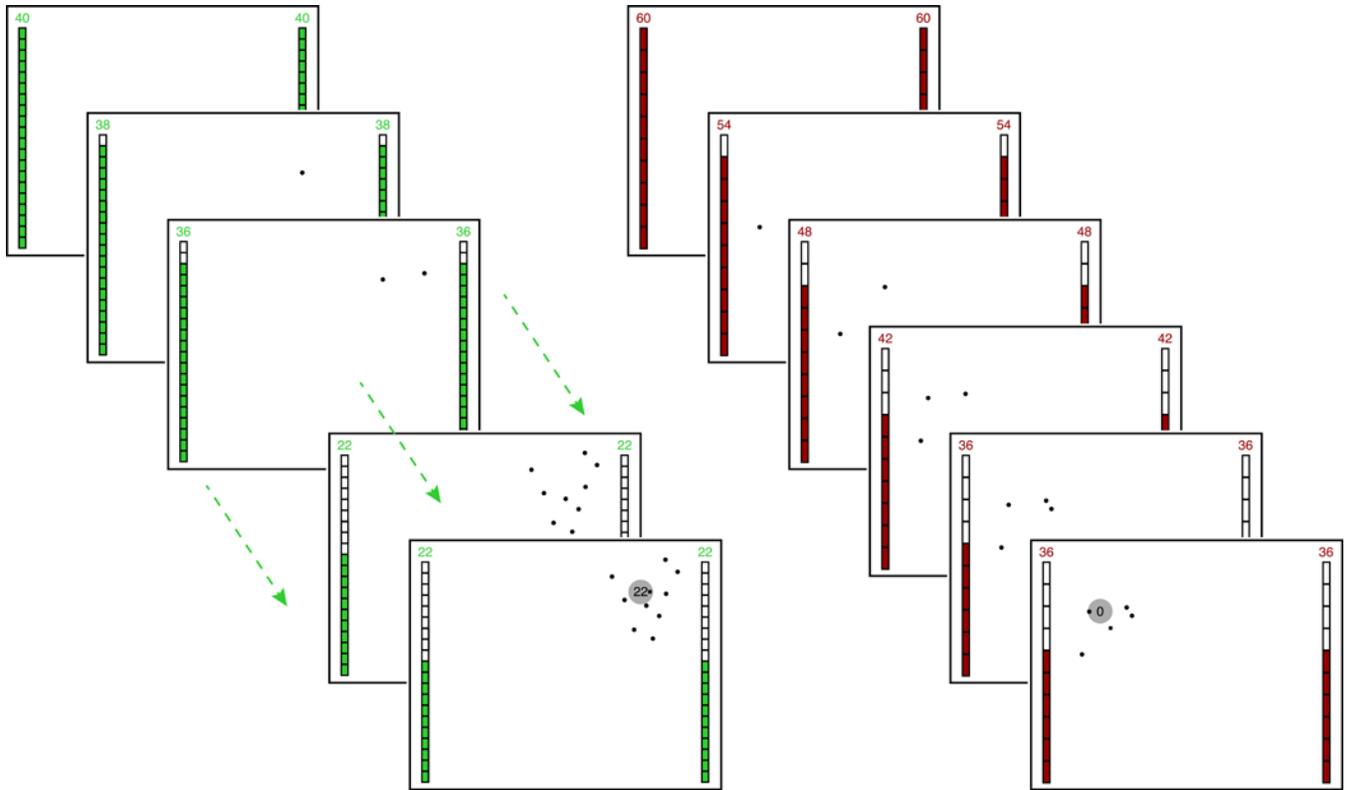


Figure 1. Schematic of the sequence of events in both experimental conditions

At the left and right edges of the screen there were identical, vertical reward bars that decreased in height with each additional cue sampled. **Left-hand panels** show an example of a low stakes trial. In this case, the participant decided to stop sampling after requesting nine cues. The potential reward for successfully touching the hidden target was reduced from 40 points to 22 points, as indicated numerically at the top of the reward bars (each cue in this condition reduces the potential reward by 2 points). The *square* dot in the bottom left panel marks the participant’s final response (which is made by touching the screen and making fine adjustments), and the hidden grey target is revealed for feedback. Since the square response dot is within the target region, the trial is coded as a hit and the reward for this trial is 22 points as indicated numerically at the center of the target. **Right-hand panels** show an example of a high stakes trial. In this case, the participant decided to stop sampling after requesting four cues, and the potential reward was reduced from 60 points to 36 points (each cue in this condition reduces the potential reward by 6 points). Since the square response dot in the bottom right panel is outside the target region, the trial is coded as a miss and there is no reward (or penalty) for this trial as indicated numerically with a 0 at the center of the target.

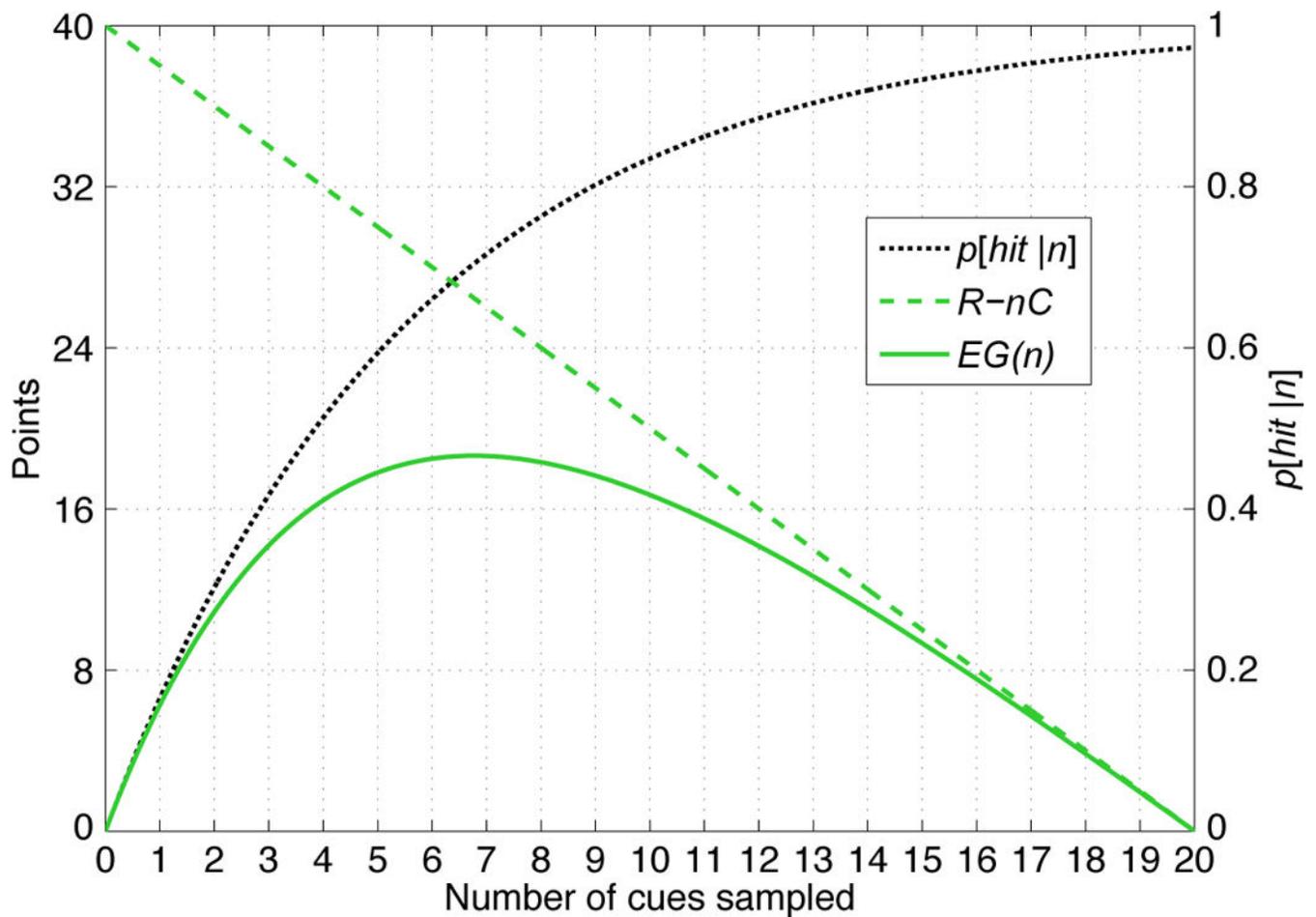


Figure 2. Expected gain function

The expected gain function $EG(n)$ is the product of the probability $p[hit|n]$ of touching the hidden target with n cues, and the gain $R-nC$ that is earned for successfully touching the target with n cues (which takes into account the cost of sampling n cues). Probability of touching the hidden target is plotted on the right vertical scale, gain and expected gain on the left. This figure shows the construction of the expected gain function for the low stakes condition. See middle panel of Figure 7A and top panel of Figure 7B for a similar construction of the expected gain function for the high stakes condition.

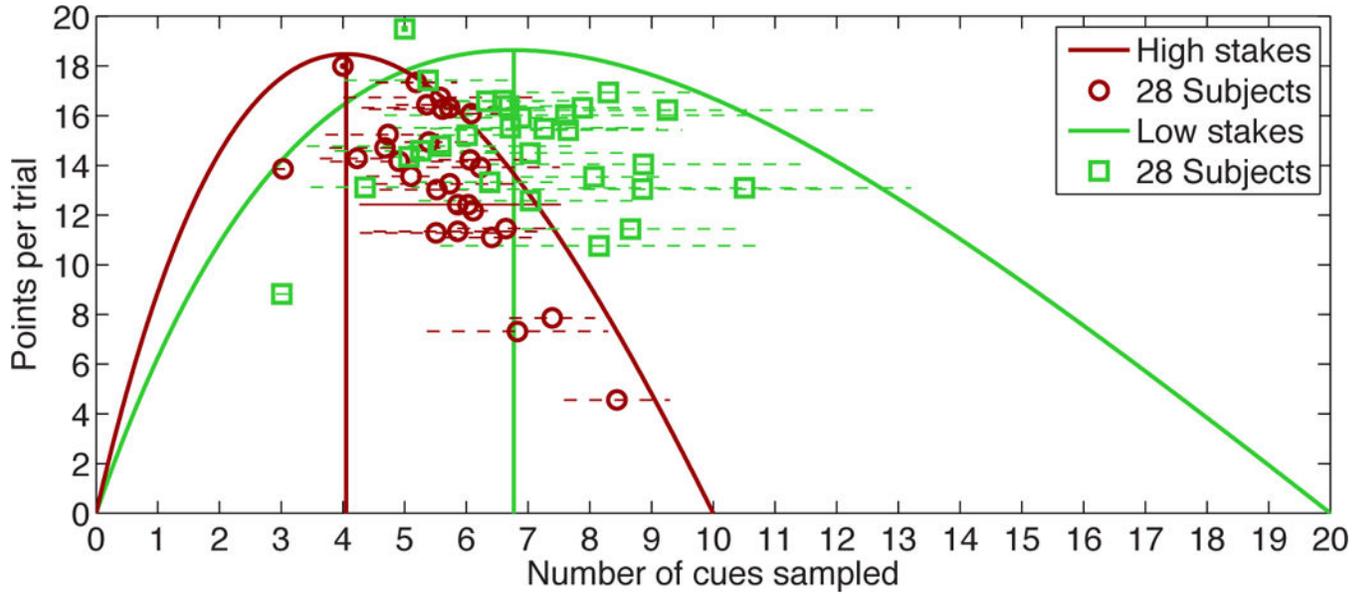


Figure 3. Results

The horizontal displacement of data points shows each participant’s mean and standard deviation of the number of cues sampled for the high stakes condition (data points shown in red in the online version of this figure, otherwise dark grey circles) and the low stakes condition (data points shown in green in the online version of this figure, otherwise light grey squares). The data points appear at heights that correspond to each participant’s average gain per trial. The expected gain curve for each condition is plotted in the corresponding color, with corresponding vertical lines marking the number of cues that the ideal participant should sample to maximize expected gain. For this figure, we plot the corresponding expected gain curves assuming that participants’ additional pointing variability $\sigma_A^2=0$. See Figure 6 and the accompanying discussion where we address the consequence that additional pointing variability (i.e., $\sigma_A^2>0$) has on the optimal sampling strategy.

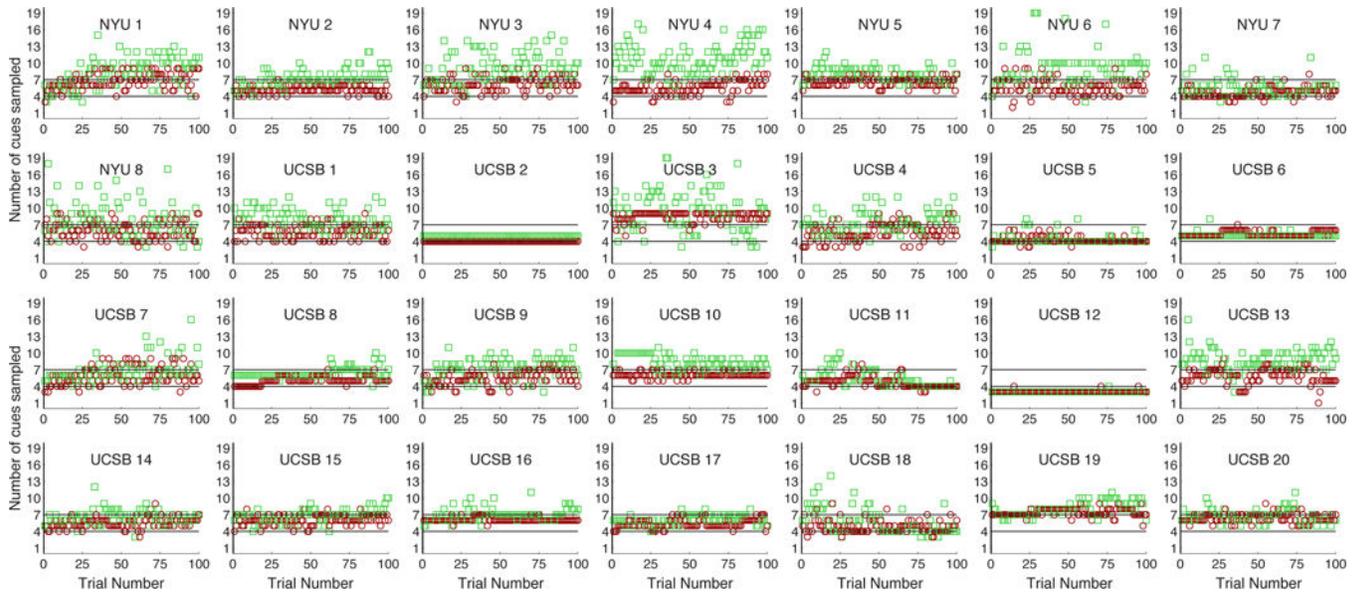


Figure 4. Trial by trial sampling behavior

Some participants (most notably UCSB 2 and UCSB 12) showed little or no variability in the number of cues that they requested across trials. Most participants however did exhibit said variability. In the main text we explore what drives this trial-to-trial variation in sampling behavior. Note that there did not seem to be any systematic learning effects as the experiment progressed towards the optimal sampling rule, which is 4 cues in the high stakes condition (data points shown in red in the online version of this figure, otherwise dark grey circles) and 7 cues in the low stakes condition ((data points shown in green in the online version of this figure, otherwise light grey squares).

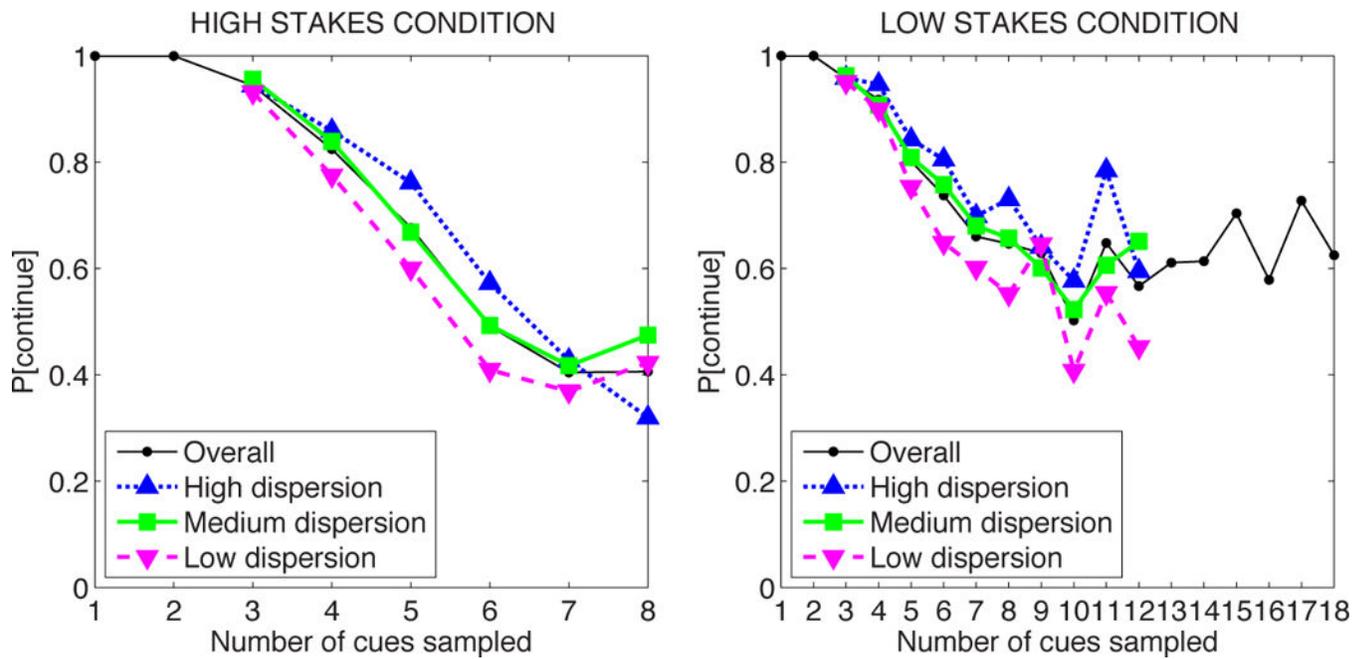


Figure 5. P[continue]

Proportion of clusters that participants chose to continue sampling in each condition is plotted as a function of (a) the sample dispersion S of the currently visible cluster and (b) the numbers of cues sampled so far in the trial. The circle data points show $P[\text{continue}]$ across all clusters irrespective of their sample dispersion. Triangles mark $P[\text{continue}]$ for clusters with high sample dispersion, squares mark $P[\text{continue}]$ for clusters with medium sample dispersion, and inverted triangles mark $P[\text{continue}]$ for clusters with low sample dispersion.

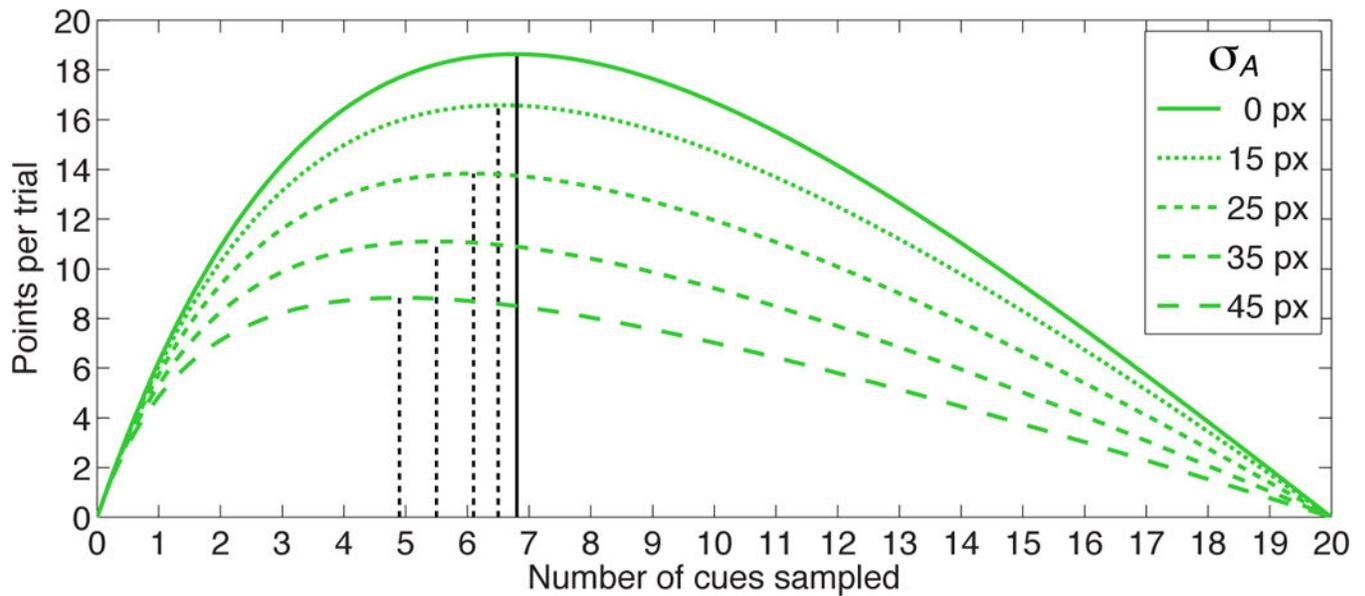


Figure 6. Effect of having additional pointing variability

The expected gain function $EG(n)$ changes with increased pointing variability (due to motor error, perceptual error, etc). Notice that as additional pointing variability increases, one would need to sample fewer and fewer cues to maximize expected gain (indicated by the solid and dashed vertical lines for different amounts of σ_A).

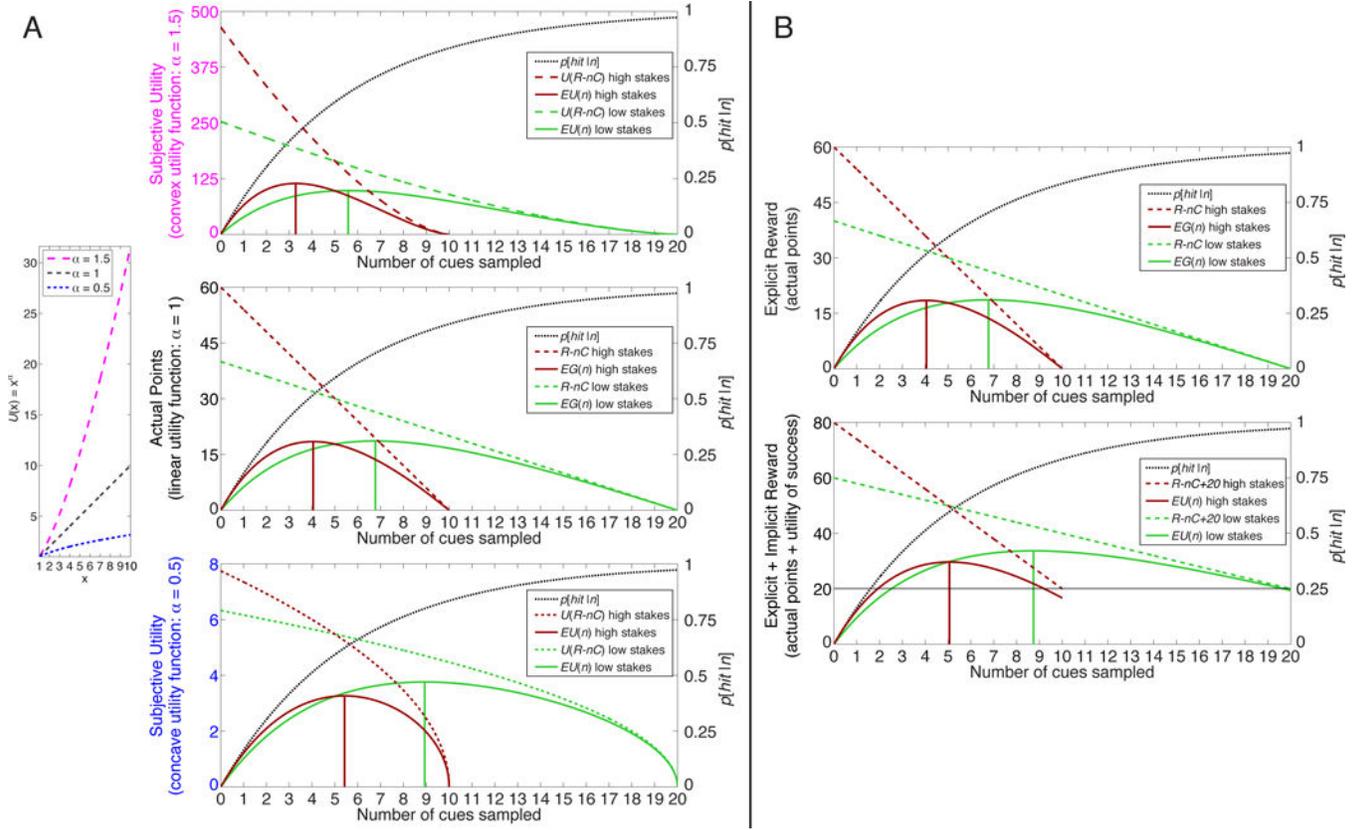


Figure 7. Effect of having a power utility function or implicit rewards
A. Power utility function. Middle panel shows the construction of the expected gain functions for both conditions when the utility function is the identity line with $\alpha=1$. Top panel shows the effect of having a convex utility function with $\alpha=1.5$. Such participants seeking to maximize their expected utility will sample fewer cues compared with a participant who has a linear utility function. Bottom panel shows the effect of having a concave utility function with $\alpha=0.5$. Such participants seeking to maximize their expected utility will sample more cues compared with a participant who has a linear utility function.
B. Implicit rewards. Top panel shows the construction of the expected gain functions for both conditions when there is no implicit reward for successfully touching the hidden target. Bottom panel shows the effect of having an implicit reward for successfully touching the hidden target that is equivalent to 20 points. Such participants seeking to maximize their *total expected explicit and implicit reward* will sample more cues compared with a participant maximizing explicit rewards only.

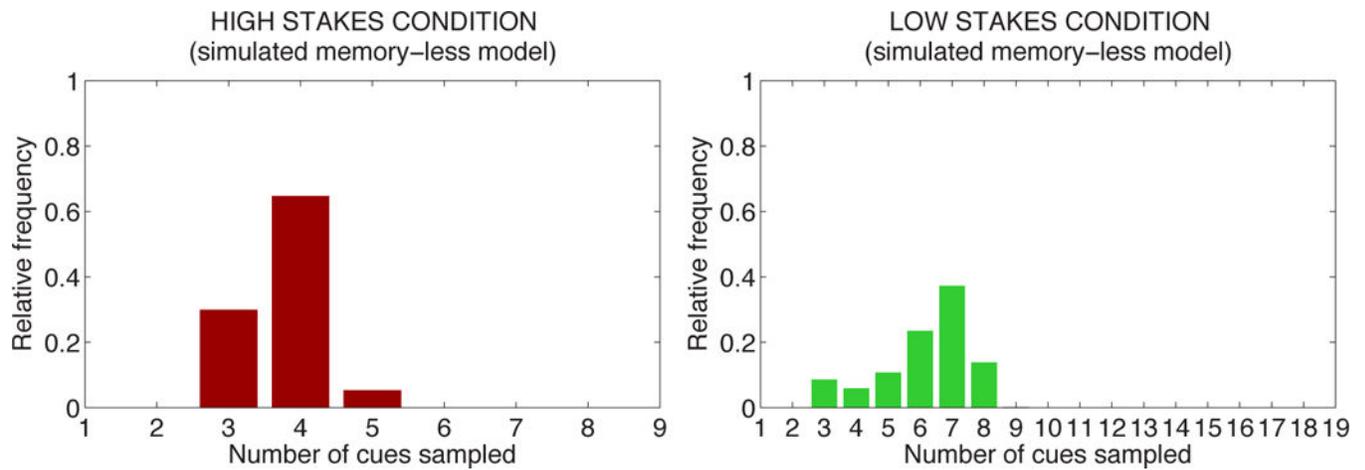


Figure 8. Hypothetical sampling behavior for a simulated memory-less participant (100,000 simulated trials per experimental condition)

The memory-less model does not accumulate information across trials about the dispersion of the underlying distribution from which the cues are drawn. Instead, the memory-less model estimates population dispersion using the sample dispersion S of the current sample. As a result, this hypothetical memory-less participant samples a variable number of cues across trials. In Table 3 we show that this hypothetical memory-less participant will sample more cues when the current cluster is more dispersed and fewer cues when it is less dispersed, a qualitative match to human sampling behavior (see Table 2).

Total respective number of clusters that participants chose to stop sampling and continue sampling as a function of the number of cues sampled so far in the trial and experimental condition.

Table 1

HIGH STAKES CONDITION		LOW STAKES CONDITION			
N	stop	continue	N	stop	continue
1	1	2799	1	0	2800
2	1	2798	2	0	2800
3	157	2641	3	120	2680
4	464	2177	4	221	2459
5	704	1473	5	488	1971
6	749	724	6	518	1453
7	431	293	7	494	959
8	174	119	8	339	620
9	119	N/A	9	230	390
			10	194	196
			11	69	127
			12	55	72
			13	28	44
			14	17	27
			15	8	19
			16	8	11
			17	3	8
			18	3	5

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HIGH STAKES CONDITION		LOW STAKES CONDITION	
stop	continue	N	stop
		19	5
			N/A

Note. We have shaded the cells for which we did not conduct our comparison analyses (cf. Table 2 and Figure 5) because of the limited number of “stop” clusters and/or “continue” clusters.

Table 2

Mean sample dispersion (in pixels) of the current cluster for “stop” clusters and “continue” clusters as a function of the number of cues sampled so far in the trial and experimental condition.

HIGH STAKES CONDITION			
N	Stop	continue	t-test
3	M=73.50 (SD=28.32)	M=75.04 (SD=27.26)	$t(2796)=0.69, p > .05$
4	M=71.65 (SD=23.43)	M=78.05 (SD=21.86)	$t(2639)=5.66, p < .001$
5	M=74.74 (SD=19.03)	M=80.72 (SD=19.03)	$t(2175)=6.86, p < .001$
6	M=77.76 (SD=16.50)	M=83.36 (SD=16.73)	$t(1471)=6.47, p < .001$
7	M=82.48 (SD=15.77)	M=84.31 (SD=15.33)	$t(722)=1.55, p > .05$
8	M=83.96 (SD=15.18)	M=82.70 (SD=11.53)	$t(291)=-0.77, p > .05$

LOW STAKES CONDITION			
N	Stop	continue	t-test
3	M=74.05 (SD=33.59)	M=74.42 (SD=27.03)	$t(2798)=0.14, p > .05$
4	M=68.99 (SD=21.99)	M=76.86 (SD=22.42)	$t(2678)=5.01, p < .001$
5	M=73.33 (SD=19.31)	M=78.23 (SD=19.52)	$t(2457)=4.99, p < .001$
6	M=74.53 (SD=17.47)	M=80.62 (SD=17.62)	$t(1969)=6.78, p < .001$
7	M=78.03 (SD=17.09)	M=81.49 (SD=15.75)	$t(1451)=3.88, p < .001$
8	M=77.89 (SD=14.16)	M=82.86 (SD=15.27)	$t(957)=4.94, p < .001$
9	M=82.19 (SD=13.79)	M=82.83 (SD=14.83)	$t(618)=0.53, p > .05$
10	M=79.94 (SD=13.74)	M=84.79 (SD=13.44)	$t(388)=3.52, p < .001$
11	M=81.15 (SD=10.47)	M=85.75 (SD=14.19)	$t(194)=2.37, p = .02$
12	M=84.26 (SD=12.78)	M=86.99 (SD=14.27)	$t(125)=1.17, p > .05$

Note. Statistical tests are shown in bold when mean sample dispersion is significantly larger for continue clusters than for stop clusters at the Bonferroni corrected p of .0031. See the Dispersion section in the Introduction for how we define and measure sample dispersion.

Table 3

Hypothetical mean sample dispersion (in pixels) of the current cluster for a simulated memory-less participant for “stop” clusters and “continue” clusters as a function of the number of cues sampled so far in the trial and experimental condition.

HIGH STAKES CONDITION (simulated memory-less model)		
N	Stop	continue
3	M=43.93 (SD=10.63)	M=88.37 (SD=21.22)
4	M=81.93 (SD=15.44)	M=126.15 (SD=10.30)
5	M=116.38 (SD=10.70)	N/A

LOW STAKES CONDITION (simulated memory-less model)		
N	stop	continue
3	M=30.71 (SD=6.72)	M=79.42 (SD=24.66)
4	M=43.94 (SD=3.89)	M=81.83 (SD=19.76)
5	M=55.06 (SD=4.12)	M=84.92 (SD=15.84)
6	M=67.84 (SD=4.95)	M=91.15 (SD=11.99)
7	M=83.43 (SD=6.61)	M=104.68 (SD=8.00)
8	M=101.31 (SD=8.04)	M=135.01 (SD=4.64)
9	M=129.78 (SD=5.13)	N/A

Note. We ran 100,000 simulated trials per experimental condition. See the Dispersion section in the Introduction for how we define and measure sample dispersion.